

Wide-band detection of the third moment of shot noise by a hysteretic Josephson junction

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We use a hysteretic Josephson junction as an on-chip detector of the third moment of shot noise of a tunnel junction. The detectable bandwidth is determined by the plasma frequency of the detector, which is about 50 GHz in the present experiment. The third moment of shot noise results in a measurable change of the switching rate when reversing polarity of the current through the noise source. We analyze the observed asymmetry assuming adiabatic response of the detector.

Studies of shot noise in mesoscopic conductors are presently of great interest, revealing detailed information on microscopic mechanisms of electronic transport [1, 2, 3]. The theory of full counting statistics (FCS) of electrons [4] determines the probability distribution of current fluctuations and its n th order moments. Experimentally, however, measurements of higher moments beyond the variance remain difficult especially in the interesting high frequency regime, mainly because of weak signals and demanding filtering requirements. The first experiments on the third moment of shot noise of a tunnel junction were performed on samples connected remotely to the detector [5, 6]. Yet the most natural way to investigate noise in nano-structures would seem to be via an on-chip detector. In recent measurements of Gustavsson et al. [7] and of Fujisawa et al. [8], single electrons tunneling through quantum dots could be counted directly. The noise correlations were observed up to the third order over a bandwidth of a few tens of kHz. The latest remarkable results on observation of the fourth and the fifth moments of current through a quantum dot [9] demonstrated on-chip detection of FCS in the low frequency regime. As regards to high-frequency on-chip detectors of non-Gaussian noise, a Josephson junction (JJ) appears to be a very attractive one according to various theoretical predictions [10, 11, 12, 13]. On the experimental side, the influence of non-Gaussian noise on the conductance of a Coulomb blocked JJ has been measured in [14]. In this Letter we present an experimental observation and a theoretical interpretation of the influence of wide-band third order fluctuations on escape of a hysteretic JJ threshold detector from its zero voltage state.

Our analysis of the experiment is based on the following model. Let us assume for a moment that the Josephson potential of the detector is strictly harmonic with oscillation (plasma) frequency $\omega_p = \sqrt{\frac{2eI_C}{\hbar C}}[1 - (I/I_C)^2]^{1/4}$, where I_C and C are the critical current and capacitance of the junction at bias current I , respectively. Then, on the level of the second moment, noise affects the escape of a JJ predominantly by promoting resonant activation at ω_p such that shot noise induced transitions between adjacent energy levels j and $j - 1$ in the po-

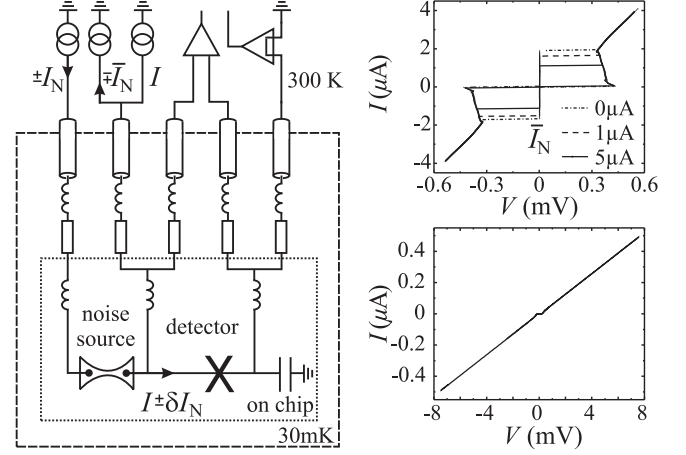


FIG. 1: Measurement scheme for on-chip detection of noise. A Josephson junction detects the current fluctuations arising from the noise source. Only heavily filtered DC lines (through cold resistors, line inductances and thermocoax cables) connect the setup with the room temperature amplifiers and current sources. On top right we show IV characteristics of the detector junction taken with different values of average current \bar{I}_N through the NIS noise source. The IV of this NIS source is depicted in the lower right graph at small currents.

tential are given by $\gamma_{j,j-1} = \frac{j}{2\hbar\omega_p C} S_I(-\omega_p)$ (excitation) and $\gamma_{j-1,j} = \frac{j}{2\hbar\omega_p C} S_I(+\omega_p)$ (relaxation), where $S_I(\omega)$ is the spectral density of noise at (angular) frequency ω . These rates lead to dynamics in the potential with effective temperature [15]

$$T^* \simeq \hbar\omega_p / [2k_B \text{arccoth}(1 + \frac{QF|\bar{I}_N|}{2I_C(1 - (I/I_C)^2)^{1/2}})]. \quad (1)$$

Here we have assumed that the bath temperature is $T \ll \hbar\omega_p/k_B$, i.e., the detector is in macroscopic quantum tunneling (MQT) regime in the absence of shot noise, and that the voltage V across the scatterer is large enough such that we are in pure shot noise regime: $|eV| \gg k_B T, \hbar\omega_p$. In Eq. (1) Q is the quality factor of the junction at plasma frequency, and \bar{I}_N is the average value of the noise current I_N through the scatterer

junction with Fano factor F [16]. Unlike in the earlier experiments by some of us [15], in the present measurements we can determine Q independently by detecting the crossover from escape dynamics to under-damped phase diffusion (UPD) [17] as described in the experimental part below. This leaves no free parameters to determine the consistency between the measured escape rate

$$\Gamma \simeq \frac{\omega_p}{2\pi} \exp(-\Delta U/k_B T^*) \quad (2)$$

and T^* given by independent measurements of the parameters in Eq. (1). In (2) $\Delta U \simeq \frac{4\sqrt{2}}{3} E_J (1 - I/I_C)^{3/2}$ is the barrier height of the potential well with Josephson energy $E_J \equiv \hbar I_C/2e$.

The linear coupling to current fluctuations δI in the Josephson potential is of the form $-\frac{\hbar}{2e} \delta I \hat{\varphi}$, where $\hat{\varphi}$ is the phase operator. It is proportional to the sum of the creation and annihilation operators of the harmonic oscillator, and therefore, in contrast to the influence of the second moment, the third order fluctuations do not induce transitions between adjacent levels in a harmonic well [12, 18]. Therefore, up to the third order, the influence of resonant activation in a harmonic potential is exclusively that due to the second order fluctuations. There are weak corrections to the results above, if we allow the potential to be anharmonic. Yet in the case of the third order fluctuations such corrections vanish unless the third order spectral densities are assumed to have frequency dependence [12, 18]. Therefore we assume in what follows that the third order effects come from non-resonant fluctuations only, at sub-plasma frequencies. Throughout we ignore the effects related to higher than third order fluctuations. Under these conditions, in analogy to adiabatic Gaussian noise [19], the contribution of the third moment to the escape rate from the well is determined by the instantaneous value of the fluctuating bias current. The asymmetry $\Delta\Gamma/\Gamma_{\text{ave}} \equiv (\langle\Gamma^+\rangle - \langle\Gamma^-\rangle)/[(\langle\Gamma^+\rangle + \langle\Gamma^-\rangle)/2]$ between escape rates $\langle\Gamma^\pm\rangle$, averaged over the adiabatic fluctuations, at different polarities of either pulse or noise currents can then be written as [12] $\Delta\Gamma/\Gamma_{\text{ave}} \simeq -\frac{1}{3}(\frac{\partial B}{\partial I})^3 \langle\delta I_N^3\rangle$. Here B is the exponent in the expression of the tunneling rate and $\langle\delta I_N^3\rangle$ is the third moment of current fluctuations at the detector. For thermal activation of Eq. (2), with $B \equiv \Delta U/k_B T^*$, one then obtains

$$\Delta\Gamma/\Gamma_{\text{ave}} \simeq \frac{16\sqrt{2}}{3} \left(\frac{\hbar}{2e}\right)^3 (1 - I/I_C)^{3/2} \frac{\langle\delta I_N^3\rangle}{(k_B T^*)^3}. \quad (3)$$

Here, $\langle\delta I_N^3\rangle \equiv (\frac{\Delta\omega}{2\pi})^2 S_3$, where $\Delta\omega \sim \omega_p$ is the bandwidth of adiabatic fluctuations, and $S_3 = F_3 e^2 \bar{I}_N$ is the low frequency limit of the third order spectral density with Fano factor F_3 .

The measurement scheme is shown in Fig. 1. The detector is an Al/AlOx/Al JJ with an area of $\sim 2 \mu\text{m}^2$

TABLE I: Parameters of the samples.

Sample	detector			noise junction	
	I_C (μA)	C (fF)	$\omega_p/2\pi$ (GHz)	R_N (k Ω)	C_N (fF)
NIS	2.88	80	53	15.4	8
SIS	2.51	80	49	1.9	6
REF	2.36	75	49	0	-

and it stays initially in a superconducting state. Another tunnel junction, the noise source, is biased with a current $\pm I_N$, such that it is driven far from equilibrium into the shot noise regime. All samples were fabricated by electron beam lithography and shadow evaporation and they were measured via filtered signal lines at a bath temperature of $\simeq 30$ mK. In order to detect only the fluctuations of I_N , the balancing current $\mp \bar{I}_N$ of opposite polarity is applied such that no DC-component due to I_N passes through the detector. Owing to the small magnitude of the third order fluctuations, careful DC-balancing is extremely important. The residual non-balanced current across the detector is measured repeatedly by a low input impedance current amplifier. However, the resulting correction never exceeded 1 nA during the measurements. The on-chip inductors in Fig. 1 are long (few mm) and narrow ($2 \mu\text{m}$) superconducting lines. In all the samples, the detector was connected to a large contact pad on the chip (few mm^2 in area), which served as a capacitive short to ground at high frequencies. To probe the fluctuations of I_N , we applied trapezoidal current pulses of height I through the detector with $\Delta t = 100 \mu\text{s}$ - 1 ms duration, and with a 1 ms delay between two pulses. Typically $10^3 - 10^4$ pulses at each value of I were repeated, and the escape probability P was obtained as the fraction of pulses leading to an escape from the supercurrent state. When increasing the average value of I_N , shot noise enhances the escape rate, leading to suppression of the escape threshold current [15]. To extract information about the third moment, we applied four different combinations of current polarities, which we call I^{++} , I^{--} , I^{+-} , and I^{-+} , where the superscripts refer to the signs of I and I_N , respectively. The first two combinations should lead to identical escape characteristics, which should differ, in the presence of odd moments, from the last two identical combinations. We normally measure the separation of the histograms, $I^+ - I^-$, corresponding to equal escape probability P in the two pulse/noise current directions. By a simple geometric argument, assuming linear dependence between escape probability and current over a short interval (weak third order effects), we obtain

$$I^+ - I^- \simeq (1 - P) \ln(1 - P) \left(\frac{\partial I}{\partial P}\right) \Delta\Gamma/\Gamma_{\text{ave}}. \quad (4)$$

We present data of three samples, see Table I. Sample NIS had a normal metal-insulator-superconductor Cu/AlOx/Al noise source. In Sample SIS the noise source was another Al/AlOx/Al JJ, but with a much

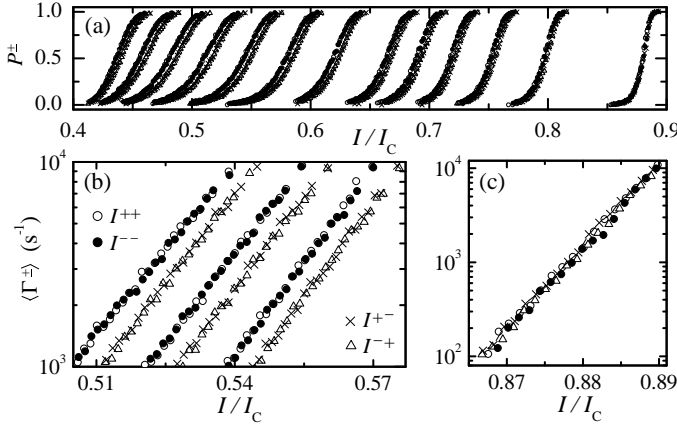


FIG. 2: Escape characteristics of Sample NIS. (a) Histograms taken with the four different polarity configurations of noise and pulse currents, and at several values of noise current: $|\bar{I}_N| = 0 \dots 3.8 \mu\text{A}$ from right to left. (b) Escape rates corresponding to $|\bar{I}_N| = 2.2, 2.4$, and $2.6 \mu\text{A}$ from right to left displaying the splitting of data with I^{++} and I^{--} currents from those with I^{+-} and I^{-+} currents, respectively. (c) Escape rates with $\bar{I}_N = 0$ in MQT regime.

smaller critical current than in the detector. In the reference sample, Sample REF, the noise source was replaced by a superconducting line. In Table I R_N denotes the normal state resistance of the noise junction, and ω_p is taken at $I = 0$.

Here we describe data of Sample NIS unless otherwise specified. Figure 1 shows current-voltage characteristics of the detector and the noise source junction. The detector IV s have been measured at different values of noise current. Besides the standard hysteretic character and suppression of switching threshold upon increasing \bar{I}_N , there are two important features to observe here. (i) The IV s are vertical in the supercurrent branch, such that $V \leq 0.2 \mu\text{V}$ up to the switching current. This means that the rate of phase diffusion events from one well to another is $f_{2\pi} = 2eV/h \leq 100 \text{ MHz}$. (ii) The gap voltage is the same at all values of noise current up to $5 \mu\text{A}$; this is higher than all those currents used for further analysis of data. Thus there is no excess overheating of the bath and the only relevant temperature is then T^* . Figure 2 shows escape characteristics with rates $\langle \Gamma^\pm \rangle$ that are related to the measured escape probabilities P^\pm via $P^\pm = 1 - e^{-\langle \Gamma^\pm \rangle \Delta t}$. Here the current pulses were $\Delta t = 480 \mu\text{s}$ long. The measurements were performed at various values of \bar{I}_N using the four polarity configurations. Suppression of the mean bias current and change in the width with increasing \bar{I}_N is evident from the histograms in Fig 2 (a). Moreover, as to the different polarity configurations, the I^{++} and I^{--} data indeed lie together for the same magnitude of \bar{I}_N , and the suppression of switching threshold for this pair is higher than for

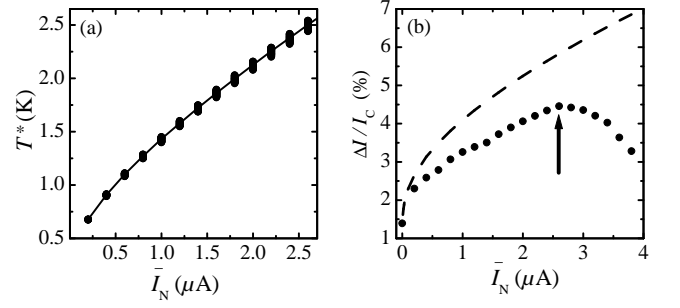


FIG. 3: Results related to the second moment of shot noise. (a) Effective temperature extracted from measured escape histograms using Eq. (2). At each noise current T^* has been evaluated at various values of escape probability in the range 0.1 - 0.9. The solid line is obtained using Eq. (1) with $F = 1$ and $Q = 2.5$ determined from the bias current at which the underdamped phase diffusion sets in. This is shown by the arrow in (b), where width of the histogram has been plotted against \bar{I}_N . The dashed line is from the effective temperature model, ignoring phase diffusion.

I^{+-} and I^{-+} . The latter two are again grouped together, as depicted in Fig. 2 (b). This shift is the manifestation of the skewness of the current distribution, where the sign of the third moment is positive. The values of I_C and C of the detector in Table I are obtained from fits to MQT escape [20]; Fig. 2 (c) shows the rates. The obtained value of I_C agrees with the Ambegaokar-Baratoff prediction [21], and C with that based on the size of the junction.

In Fig. 3 we have analyzed the escape histograms of Fig. 2 based on Eq. (2) to extract effective temperature at various levels of \bar{I}_N . We obtain in (a) a set of values of T^* , spread within about $\pm 2\%$ around the mean, for each \bar{I}_N at different values of bias current I along a histogram. For clarity, we show data of one pulse configuration (I^{+-}) only, because the third order effects are weak: the variation in T^* between different configurations is only about 1% at maximum. Figure 3 (b) shows the measured width, defined as ΔI between $P = 0.9$ and $P = 0.1$ of the same escape histograms. We see an increase up to about $\bar{I}_N = 2.6 \mu\text{A}$, in fair agreement with the thermal activation model (dashed line). Above this value of \bar{I}_N the width starts to decrease. We attribute the maximum to the crossover from escape dynamics to UPD [17], now induced by the increase of effective temperature and subsequent decrease of the switching threshold current. From the value of $I/I_C \simeq 0.52$ for the $P = 0.5$ switching threshold at this noise current, we obtain the quality factor $Q \equiv \frac{4I_C}{\pi T} \simeq 2.5$. (Similar procedure gave $Q = 2.5$ also for Sample SIS.) Based on this value of Q and the values of I_C and C from independent measurements, we are then able to draw the theoretical line based on Eq. (1) in Fig. 3 (a).

Figure 4 shows the influence of the odd moments on

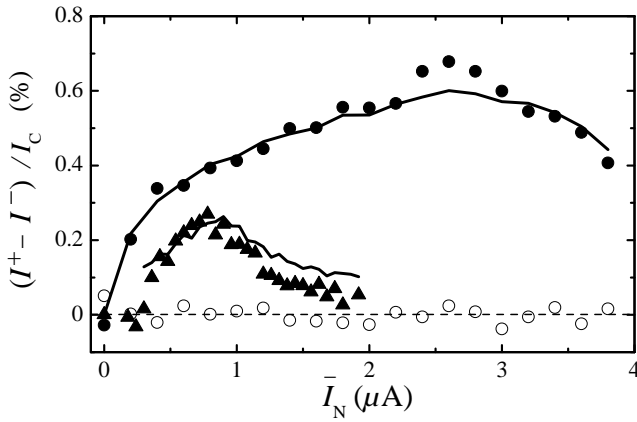


FIG. 4: Difference in switching currents of the threshold detector under reversal of the relative polarities of bias and noise currents. Data of Sample NIS are shown by filled dots, of Sample SIS by triangles and data of the reference sample, Sample REF, by open dots. The solid lines are the result of the theoretical model for Samples NIS and SIS. At currents $\bar{I}_N \leq 0.2 \mu\text{A}$ the noise source of Sample SIS is in superconducting state, and the signal due to the third moment vanishes.

escape threshold of all the three samples. The quantity $I^+ - I^-$ has been extracted as $\frac{1}{2}[(I^{+-} + I^{-+}) - (I^{++} + I^{--})]$, all taken at $P = 0.5$. We see that the signal due to the third moment, plotted against \bar{I}_N again, of Sample NIS (filled dots) has a maximum value of about 0.6% of I_C . This is to be contrasted to the essentially vanishing signal of the reference sample (open circles). The solid line following closely the data of Sample NIS is the result of the theoretical model according to Eqs. (3) and (4). The non-trivial parameters needed to construct this curve, $\partial I / \partial P$ and T^* , have been extracted from experiment. Note that values of F and Q are not needed here since we measure T^* directly. In addition, we have set $F_3 = 1$, the value for Poissonian noise of a tunnel junction, and $\Delta\omega = \omega_p$, which is a very natural choice for our model including adiabatic frequencies only. We also show similar data of Sample SIS (triangles). Here, the bandwidth was taken to be $0.8\omega_p$ for the theory line. It may seem surprising at the first view that the results follow the theoretical prediction also in the regime where the detector is in the UPD regime ($\bar{I}_N > 2.6 \mu\text{A}$ for Sample NIS). Yet this is quite natural, because the frequency of the phase diffusion events is $< 100 \text{ MHz}$, whereas the relevant bandwidth of noise affecting the phase particle extends up to $\sim 50 \text{ GHz}$. Therefore it suffices to study escape from one well only.

We have observed the third moment of shot noise up to frequencies of $\sim 50 \text{ GHz}$, and interpreted this observation by assuming that the third order effects manifest themselves in our JJ system as an average thermal escape rate in a potential which varies according to adi-

abatically fluctuating bias currents. A similar problem has been analyzed theoretically in [11] using an effective Fokker-Planck method and very recently in [13] by generalized stochastic path integral methods. The results of our model coincide with [11] up to numerical prefactor of the order of unity in the limit of large Q , and with [13] excluding the circuit corrections that are weak in our case. Finally, although the data presented here can be interpreted quantitatively, it remains a challenge to engineer the radio-frequency environment on the chip to the degree that no uncertainty in the relevant bandwidth would remain. However, it is already possible to envision experiments where different mesoscopic noise sources use a common JJ detector of third order fluctuations, and where a tunnel junction serves as a reference scatterer.

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